Assignment 1

Due date: Friday, February 3

Gyrating ring

A ring of radius R rolls without slipping on a table while its axis is inclined with respect to the vertical by angle α. As the ring gyrates, its center of mass stays fixed in space and its point of contact with the table moves around a circle with period T. Obtain an explicit expression for the ring's angular velocity vector $\boldsymbol{\omega}$.

Rolling sphere

A sphere of radius R rolls without slipping on a table. Interpret "rolling without slipping" in this case as the property that the point of the sphere making contact with the table is instantaneously at rest. Obtain a vector relationship between the sphere's angular velocity $\boldsymbol{\omega}$, linear velocity $\mathbf{v}$, and the unit normal vector of the table, $\mathbf{n}$.

The number of "degrees of freedom" is usually defined as the number of continuous parameters required to uniquely specify the positions in a mechanical system. A better definition counts the number of independent velocity components of the system's *motion*. According to the better definition, how many degrees of freedom does a rolling ball have?

Time dependent angular velocity

The body frame (of some body) is related to the space frame by the orthogonal matrix U. We learned that the time rate of change of U satisfies the equation $\dot{U} = AU$, where

$$A = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

is an antisymmetric matrix whose nonzero elements correspond to the components of the angular velocity vector in the space frame. Consider a situation, as in the gyrating ring problem, where $\boldsymbol{\omega}$ has the following time-dependence:

$$\omega_x = (\omega \cos\alpha)\cos\Omega t \qquad \omega_y = (\omega\cos\alpha)\sin\Omega t \qquad \omega_z = \omega\sin\alpha.$$

Suppose the space and body frames coincide at $t = 0$, so $U(0)$ is the identity matrix. Take a period of time $T = 2\pi/\Omega$, so $\boldsymbol{\omega}$ completes one period. The rotation matrix $U(T)$ is now the net rotation of the body after the angular velocity has completed one orbit.

Write a simple computer program to compute $U(T)$ by implementing a finite-difference integration of the equation $\dot{U} = AU$:

$$U(t + \Delta t) - U(t) = \Delta t A(t)U(t).$$

- Do *not* use a canned differential equation solver; program this from scratch as a finite-difference iteration.
- Check that your final $U(T)$ is nearly orthogonal; if not, you need to decrease Δt.
- Output the resulting $U(T)$ for the case of the gyrating ring (where ω and Ω have a special relationship) with $\alpha = \pi/4$. Check your answer by calculating $U(T)$ directly, by comparing the circumference of the ring with the circumference of the ring of contact on the table (along which there is no slipping).